THE ONE-SON POLICY

1 Simulate the growth of a family:
Toss a coin ("head" for son, "tail" for girl) until a son is born and note down how many children there are. Repeat the procedure 50 times and calculate
a) the average number of children in a family.
b) the average number of girls and boys in a family.

throw	result	throw	result	throw	result	throw	result	throw	result
1	1	11	2	21	3	31	1	41	3
2	2	12	5	22	4	32	2	42	1
3	1	13	1	23	1	33	5	43	2
4	1	14	3	24	6	34	2	44	3
5	1	15	4	25	1	35	3	45	1
6	2	16	3	26	1	36	1	46	4
7	1	17	2	27	3	37	1	47	1
8	1	18	3	28	1	38	1	48	2
9	3	19	1	29	7	39	1	49	1
10	2	20	2	30	3	40	1	50	2

a) The average number of children in a family

$$= \frac{\text{sum of all children}}{50} = \frac{106}{50} \approx \mathbf{2.12}$$

b) The average number of boys in a family = **1**

$$\text{The average number of girls in a family} = \frac{\text{sum of all girls}}{50} = \frac{56}{50} \approx \mathbf{1.12}$$

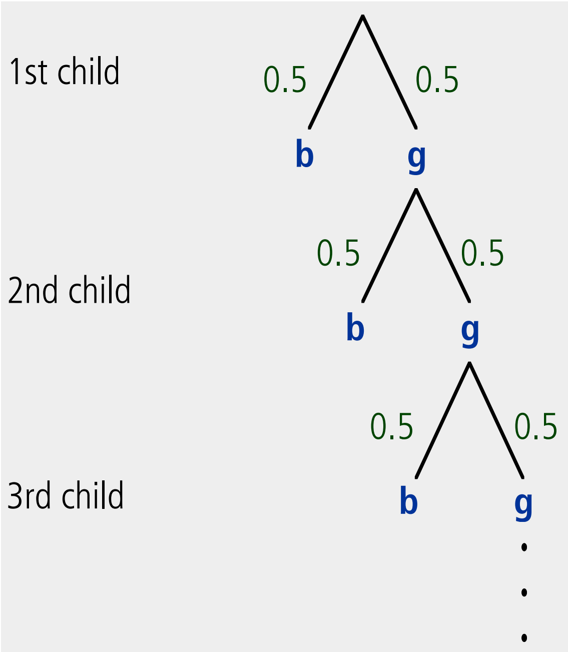
2 Calculate theoretically
a) the average number of children in a family.
b) the average number of girls and boys in a family.

a) Each birth forms a new level in a compound experiment.

b = "boy is born"
g = "girl is born"

The following families can develop:
b, gb, ggb, ...

Let X be the number of children in these families. It is the expected value of X that has to be calculated.



$$\Omega = \{1, 2, 3, \ldots\}$$

$$p(1) = \frac{1}{2}$$

$$p(2) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

$$p(3) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$$ and so on.

In workshop 12 there will be the mathematical definition of **the expected value E**.

DEFINITION

Let X be a probability experiment with n numbers as outcomes: $\Omega = \{x_1, x_2, \ldots, x_n\} \subset \mathbf{R}$

Then the **expected value** $E(X) = \mu = \sum_{i=1}^{n} p(x_i) \cdot x_i$.

$$E(X)=\frac{1}{2}\cdot 1+\frac{1}{4}\cdot 2+\frac{1}{8}\cdot 3+\frac{1}{16}\cdot 4+\ldots=\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\ldots \qquad ?$$

$$+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\ldots$$

$$+\frac{1}{8}+\frac{1}{16}+\ldots$$

$$+\frac{1}{16}+\ldots$$

$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\ldots$ is the sum of a geometric series (a_n) with $a_1=\frac{1}{2}$ and

$r=\frac{1}{2}$. $\Rightarrow$ $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\ldots=\frac{\frac{1}{2}}{1-\frac{1}{2}}=\frac{\frac{1}{2}}{\frac{1}{2}}=1$

$\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\ldots$ is the sum of an geometric series (b_n) with $b_1=\frac{1}{4}$ and $r=\frac{1}{2}$.

$$\Rightarrow \quad \frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\ldots=\frac{\frac{1}{4}}{1-\frac{1}{2}}=\frac{\frac{1}{4}}{\frac{1}{2}}=\frac{1}{2}$$

$\frac{1}{8}+\frac{1}{16}+\ldots$ is the sum of an geometric series (c_n) with $c_1=\frac{1}{8}$ and $r=\frac{1}{2}$.

$$\Rightarrow \quad \frac{1}{8}+\frac{1}{16}+\ldots=\frac{\frac{1}{8}}{1-\frac{1}{2}}=\frac{\frac{1}{8}}{\frac{1}{2}}=\frac{1}{4}$$

and so on.

$\Rightarrow$ $E(X)=\frac{1}{2}\cdot 1+\frac{1}{4}\cdot 2+\frac{1}{8}\cdot 3+\frac{1}{16}\cdot 4+\ldots=1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots$ is another GS (e_n) with

$e_1=1$ and $r=\frac{1}{2}$ $\quad\Rightarrow\quad$ $E(X)=\frac{1}{1-\frac{1}{2}}=\mathbf{2}$

b) There is always exactly **1 boy** in every family.
If there are 2 children in the average family and if there is 1 boy in the average family there must be **1** girl in the average family, too!